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Neutrosophic Vague Soft Set and its Applications

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ABSTRACT

The notion of classical soft sets is extended to neutrosophic vague soft sets by applying the theory of soft sets to neutrosophic vague sets to be more effective and useful. We also define its null, absolute and basic operations, namely complement, subset, equality, union and intersection along with illustrative examples, and study some related properties with supporting proofs. Lastly, this notion is applied to a decision making problem and its effectiveness is demonstrated using an illustrative example.

Keywords: Decision making, neutrosophic set, neutrosophic soft set, soft set, vague set.

1. Introduction

Fuzzy set was introduced by Zadeh (1965) as a mathematical tool to solve problems and vagueness in everyday life. Since then the fuzzy sets and fuzzy logic have been applied in many real life problems in uncertain, ambiguous environment by Ahmadian et al. (2016), Paul and Bhattacharya (2015), Ramli and Mohamad (2014) and Ramli et al. (2014). A great deal of research were undertaken to deal with uncertainty, such as by Pawlak (1982), Gau and Buehrer (1993) and Atanassov (1986), and recently neutrosophic set theory (Smarandache (2005)) as a generalization of the intuitionistic set, classical set, fuzzy set, paraconsistent set, dialetheist set, paradoxist set, tautological set based on "neutrosophy". The words "neutrosophy" and "neutrosophic" were introduced by Smarandache in his 1998 book (Smarandache (1998)). "neutrosophy" (noun) means knowledge of neutral thought, while "neutrosophic" (adjective), means having the nature of, or having the characteristic of neutrosophy. Molodtsov (1999) firstly proposed soft set theory to cope with uncertainty and vagueness. Since then, many research in the literature were undertaken on soft set and fuzzy set, which incorporates the beneficial properties of both soft set and fuzzy set techniques, by Alkhazaleh et al. (2011a), Salleh et al. (2012), Maji et al. (2001a), Adam and Hassan (2014a, 2015, 2016a,b, 2017), Feng et al. (2010) and Zhang and Zhang (2012). In turn, many research were undertaken on intuitionistic fuzzy soft set and vague soft set by Maji et al. (2001b) , Alhazaymeh et al. (2012) and Xu et al. (2010). Neutrosophic soft set was then introduced (Maji (2013)). Moreover, soft set has been developed rapidly to soft expert set (Alkhazaleh and Salleh (2011)), fuzzy parameterized single valued neutrosophic soft expert set theory (Al-Quran and Hassan (2016a)), soft multiset theory (Alkhazaleh et al. (2011b)), Q-fuzzy soft sets (Adam and Hassan (2014b,c,d,e)), vague soft sets relations and functions (Alhazaymeh and Hassan (2015)), intuitionistic neutrosophic soft set (Broumi and Smarandache (2013)), interval-valued neutrosophic soft sets and its decision making (Deli (2015)), interval-valued vague soft sets and its applications (Alhazaymeh and Hassan (2012b, 2013a,b, 2014a)), thereby opening avenues to many applications such as variants of vague soft set (Hassan and Alhazaymeh (2013); Hassan and Al-Quran (2017)), and its application in decision making (Alhazaymeh and Hassan (2012a,c, 2014b, 2017a,b); Al-Quran and Hassan (2016b)), genetic algorithm (Varnamkhasti and Hassan (2012, 2013, 2015)), and on similarity and entropy of neutrosophic soft sets (Sahin and Kucuk (2014)).

Alkhazaleh (2015) introduced the concept of neutrosophic vague set as a combination of neutrosophic set and vague set. Neutrosophic vague theory is an effective tool to process incomplete, indeterminate and inconsistent information. Now, suppose we are to develop the neutrosophic vague set to be more effective and useful to solve decision making problems. The question is: Can we construct a parameterization tool that represents the parameters of the neutrosophic vague set in a complete and comprehensive way? This paper aims to answer this question by deriving an improved hybrid model of neutrosophic vague set and soft set. This improved model is called neutrosophic vague soft set (NVSS). There are four main contributions of this paper. Firstly, we introduce neutrosophic vague soft set (NVSS) which deals with incompleteness, impreciseness, uncertainty and indeterminacy. Secondly, we define null, absolute and basic operations, namely complement, subset, equality, union and intersection along with illustrative examples, and study some related properties with supporting proofs. Thirdly, we give an algorithm for decision making using neutrosophic vague soft sets. Lastly, we give the comparison of neutrosophic vague soft sets to the current methods.

The organization of this paper is as follows. We first present the basic definitions of neutrosophic vague set, neutrosophic set and neutrosophic soft set theory that are useful for subsequent discussions. We then propose a novel concept of neutrosophic vague soft set which is a combination of neutrosophic vague set and soft set to improve the reasonability of decision making in reality. We also define null, absolute and basic operations, namely subset, equality, complement, union and intersection along with several examples, and study their properties. Finally we give a decision making method for neutrosophic vague soft set theory and present an application of this concept in solving a decision making problem. Thus decision making are not necessarily be in discrete terms (Hassan and Ayop (2012) , Hassan and Halim (2012), Hassan and Loon (2012), Hassan and Sahrin (2012) , Hassan et al. (2012), Hassan and Tabar (2011), Hassan et al. (2010a,b)).

2. Preliminaries

In this section, we recall some basic notions in neutrosophic vague set, neutrosophic set and neutrosophic soft set theory.

Definition 2.1 (Smarandache (2005)) A neutrosophic set A on the universe of discourse X is defined as $A = \{< x; T_A(x); I_A(x); F_A(x) >; x \in X\}$, where $T; I; F : X \to]^-0; 1^+[$ and $^-0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+$.

Definition 2.2 (Gau and Buehrer (1993)) Let X be a space of points (objects), with a generic element of X denoted by x. A vague set V in X is characterized by a truth-membership function t_v and a false-membership function f_v . $t_v(x)$ is a lower bound on the grade of membership of x derived from the evidence for x , and $f_v(x)$ is a lower bound on the negation of x derived from the evidence against x . $t_v(x)$ and $f_v(x)$ both associate a real number in the interval $[0, 1]$ with each point in X, where $t_v(x) + f_v(x) \leq 1$.

Definition 2.3 (Alkhazaleh (2015)) A neutrosophic vague set A_{NV} (NVS in short) on the universe of discourse X can be written as

$$A_{NV} = \{< x; \widehat{T}_{A_{NV}}(x); \widehat{I}_{A_{NV}}(x); \widehat{F}_{A_{NV}}(x) >; x \in X\}$$

whose truth-membership, indeterminacy - membership and falsity-membership functions is defined as $\widehat{T}_{A_{NV}}(x) = [T^-, T^+]$, $\widehat{I}_{A_{NV}}(x) = [I^-, I^+]$ and $\widehat{F}_{A_{NV}}(x) = [F^-, F^+]$, where

1. $T^+ = 1 - F^-$, $F^+ = 1 - T^-$ and 2. ${}^-0 \leq T^- + I^- + F^- \leq 2^+$.

Definition 2.4 (Alkhazaleh (2015)) Let A_{NV} and B_{NV} be two NVSs of the universe U. If $\forall u_i \in U$,

1. $\widehat{T}_{A_{NV}}(u_i) = \widehat{T}_{B_{NV}}(u_i)$, 2. $\widehat{I}_{A_{NV}}(u_i) = \widehat{I}_{B_{NV}}(u_i)$, and 3. $\widehat{F}_{A_{NV}}(u_i) = \widehat{F}_{B_{NV}}(u_i)$, then the NVS A_{NV} and B_{NV} are called equal, where $1 \leq i \leq n$.

Definition 2.5 (Alkhazaleh (2015)) Let A_{NV} and B_{NV} be two NVSs of the universe U. If $\forall u_i \in U$,

1. $\widehat{T}_{A_{NV}}(u_i) \leq \widehat{T}_{B_{NV}}(u_i)$, 2. $\widehat{I}_{A_{NV}}(u_i) \geq \widehat{I}_{B_{NV}}(u_i)$, and 3. $\widehat{F}_{A_{NV}}(u_i) \geq \widehat{F}_{B_{NV}}(u_i)$, then the NVS A_{NV} are included by B_{NV}, denoted by $A_{NV} \subseteq B_{NV}$, where $1 \leq i \leq n$.

Definition 2.6 (Alkhazaleh (2015)) The complement of a NVS A_{NV} is denoted by A^c and is defined by

$\widehat{T}^c_{A_{NV}}(x) = [1 - T^+, 1 - T^-]$, $\widehat{I}^c_{A_{NV}}(x) = [1 - I^+, 1 - I^-]$, and
$\widehat{F}^c_{A_{NV}}(x) = [1 - F^+, 1 - F^-]$.

Definition 2.7 (Alkhazaleh (2015)) The union of two NVSs A_{NV} and B_{NV} is a NVS C_{NV}, written as $C_{NV} = A_{NV} \cup B_{NV}$, whose truth-membership, indeterminacy-membership and false-membership functions are related to those of A_{NV} and B_{NV} given by

$$\widehat{T}_{C_{NV}}(x) = \left[\max\left(T^{-}_{A_{NV\,x}}, T^{-}_{B_{NV\,x}}\right), \max\left(T^{+}_{A_{NV\,x}}, T^{+}_{B_{NV\,x}}\right)\right],$$
$$\widehat{I}_{C_{NV}}(x) = \left[\min\left(I^{-}_{A_{NV\,x}}, I^{-}_{B_{NV\,x}}\right), \min\left(I^{+}_{A_{NV\,x}}, I^{+}_{B_{NV\,x}}\right)\right] \text{ and}$$
$$\widehat{F}_{C_{NV}}(x) = \left[\min\left(F^{-}_{A_{NV\,x}}, F^{-}_{B_{NV\,x}}\right), \min\left(F^{+}_{A_{NV\,x}}, F^{+}_{B_{NV\,x}}\right)\right].$$

Definition 2.8 (Alkhazaleh (2015)) The intersection of two NVSs A_{NV} and B_{NV} is a NVS C_{NV}, written as $H_{NV} = A_{NV} \cap B_{NV}$, whose truth-membership, indeterminacy-membership and false-membership functions are related to those of A_{NV} and B_{NV} given by

$$\widehat{T}_{H_{NV}}(x) = \left[\min\left(T^{-}_{A_{NV\,x}}, T^{-}_{B_{NV\,x}}\right), \min\left(T^{+}_{A_{NV\,x}}, T^{+}_{B_{NV\,x}}\right)\right],$$
$$\widehat{I}_{H_{NV}}(x) = \left[\max\left(I^{-}_{A_{NV\,x}}, I^{-}_{B_{NV\,x}}\right), \max\left(I^{+}_{A_{NV\,x}}, I^{+}_{B_{NV\,x}}\right)\right] \text{ and}$$
$$\widehat{F}_{H_{NV}}(x) = \left[\max\left(F^{-}_{A_{NV\,x}}, F^{-}_{B_{NV\,x}}\right), \max\left(F^{+}_{A_{NV\,x}}, F^{+}_{B_{NV\,x}}\right)\right].$$

3. Neutrosophic Vague Soft Set (NVSS)

In this section, we introduce the definition of a neutrosophic vague soft set and define some operations on this concept, namely equality, subset hood, absolute and null NVSS.

Definition 3.1 Let U be a universe , E a set of parameters and $A \subseteq E$. A collection of pairs $(\widehat{F}, A)$ is called a *neutrosophic vague soft set* (NVSS) over U where $\widehat{F}$ is a mapping given by

$$\widehat{F} : A \to NV(U),$$

and $NV(U)$ denotes the set of all neutrosophic vague subsets of U .

Example 3.2 Let $U = \{t_1, t_2, t_3, t_4\}$ be a set of universe representing residential buildings. Let $A = \{e_1, e_2, e_3\} = \{large, small, medium\}$ be a set of parameters defining the size of the residence. Define a mapping

$$\widehat{F} : A \to NV(U),$$

as :

$$\widehat{F}(e_1)=\left\{\frac{t_1}{\langle[0.2,0.8];[0.1,0.3];[0.2,0.8]\rangle},\frac{t_2}{\langle[0.1,0.7];[0.2,0.5];[0.3,0.9]\rangle},\frac{t_3}{\langle[0.5,0.6];[0.3,0.7];[0.4,0.5]\rangle},\frac{t_4}{\langle[0.8,1];[0.1,0.2];[0,0.2]\rangle}\right\},$$

$$\widehat{F}(e_2)=\left\{\frac{t_1}{\langle[0.8,0.9];[0.3,0.4];[0.1,0.2]\rangle},\frac{t_2}{\langle[0.2,0.4];[0.2,0.4];[0.6,0.8]\rangle},\frac{t_3}{\langle[0,0.5];[0.5,0.7];[0.5,1]\rangle},\frac{t_4}{\langle[0.6,0.7];[0.2,0.4];[0.3,0.4]\rangle}\right\},$$

$$\widehat{F}(e_3)=\left\{\frac{t_1}{\langle[0.6,0.9];[0.2,0.4];[0.1,0.4]\rangle},\frac{t_2}{\langle[0.7,0.8];[0.3,0.5];[0.2,0.3]\rangle},\frac{t_3}{\langle[0.6,0.8];[0.1,0.4];[0.2,0.4]\rangle},\frac{t_4}{\langle[0.2,0.4];[0.5,0.6];[0.6,0.8]\rangle}\right\}.$$

Then we can write the neutrosophic vague soft set $\left(\widehat{F},A\right)$ as the following collection of approximations:

$$\begin{aligned}\left(\widehat{F},A\right)=\Bigg\{&\left(e_1,\left\{\frac{t_1}{\langle[0.2,0.8];[0.1,0.3];[0.2,0.8]\rangle},\frac{t_2}{\langle[0.1,0.7];[0.2,0.5];[0.3,0.9]\rangle},\frac{t_3}{\langle[0.5,0.6];[0.3,0.7];[0.4,0.5]\rangle},\frac{t_4}{\langle[0.8,1];[0.1,0.2];[0,0.2]\rangle}\right\}\right),\\&\left(e_2,\left\{\frac{t_1}{\langle[0.8,0.9];[0.3,0.4];[0.1,0.2]\rangle},\frac{t_2}{\langle[0.2,0.4];[0.2,0.4];[0.6,0.8]\rangle},\frac{t_3}{\langle[0,0.5];[0.5,0.7];[0.5,1]\rangle},\frac{t_4}{\langle[0.6,0.7];[0.2,0.4];[0.3,0.4]\rangle}\right\}\right),\\&\left(e_3,\left\{\frac{t_1}{\langle[0.6,0.9];[0.2,0.4];[0.1,0.4]\rangle},\frac{t_2}{\langle[0.7,0.8];[0.3,0.5];[0.2,0.3]\rangle},\frac{t_3}{\langle[0.6,0.8];[0.1,0.4];[0.2,0.4]\rangle},\frac{t_4}{\langle[0.2,0.4];[0.5,0.6];[0.6,0.8]\rangle}\right\}\right)\Bigg\}.\end{aligned}$$

We can represent the NVSS in the form of Table 1. The entries are c_{ij} corresponding to the residence t_i and the parameter e_j, where c_{ij} =(true-membership neutrosophic vague value of t_i, indeterminacy-membership neutrosophic vague value of t_i, falsity-membership neutrosophic vague value of t_i) in $\widehat{F}(e_j)$.

The tabular representation of the neutrosophic vague soft set $\left(\widehat{F},A\right)$ is as in Table 1.

Table 1: Representation of $(\widehat{F}, A)$

U	e_1	e_2	e_3
(t_1)	$\langle [0.2, 0.8] ; [0.1, 0.3] ; [0.2, 0.8] \rangle$	$\langle [0.8, 0.9] ; [0.3, 0.4] ; [0.1, 0.2] \rangle$	$\langle [0.6, 0.9] ; [0.2, 0.4] ; [0.1, 0.4] \rangle$
(t_2)	$\langle [0.1, 0.7] ; [0.2, 0.5] ; [0.3, 0.9] \rangle$	$\langle [0.2, 0.4] ; [0.2, 0.4] ; [0.6, 0.8] \rangle$	$\langle [0.7, 0.8] ; [0.3, 0.5] ; [0.2, 0.3] \rangle$
(t_3)	$\langle [0.5, 0.6] ; [0.3, 0.7] ; [0.4, 0.5] \rangle$	$\langle [0, 0.5] ; [0.5, 0.7] ; [0.5, 1] \rangle$	$\langle [0.6, 0.8] ; [0.1, 0.4] ; [0.2, 0.4] \rangle$
(t_4)	$\langle [0.8, 1] ; [0.1, 0.2] ; [0, 0.2] \rangle$	$\langle [0.6, 0.7] ; [0.2, 0.4] ; [0.3, 0.4] \rangle$	$\langle [0.2, 0.4] ; [0.5, 0.6] ; [0.6, 0.8] \rangle$

Definition 3.3 Let $\left(\widehat{F}, A\right)$ and $\left(\widehat{G}, B\right)$ be two neutrosophic vague soft sets over a universe U, we say that $\left(\widehat{F}, A\right)$ is a neutrosophic vague soft subset of $\left(\widehat{G}, B\right)$ denoted by $\left(\widehat{F}, A\right) \subseteq \left(\widehat{G}, A\right)$ if and only if

1. $A \subseteq B$, and 2. $\forall e \in A, \widehat{F}(e)$ is a neutrosophic vague subset of $\widehat{G}(e)$.

Definition 3.4 For two neutrosophic vague soft sets $\left(\widehat{F}, A\right)$ and $\left(\widehat{G}, B\right)$ over a universe U, we say that $\left(\widehat{F}, A\right)$ is equal to $\left(\widehat{G}, B\right)$ and we write $\left(\widehat{F}, A\right) = \left(\widehat{G}, B\right)$ if $\left(\widehat{F}, A\right) \subseteq \left(\widehat{G}, B\right)$ and $\left(\widehat{G}, B\right) \subseteq \left(\widehat{F}, A\right)$.

Example 3.5 Let $\left(\widehat{F}, A\right)$ and $\left(\widehat{G}, B\right)$ be two neutrosophic vague soft sets over the universe $U = \{u_1, u_2, u_3\}$.

Consider the tabular representation of the NVSS $\left(\widehat{F}, A\right)$ as in Table 2.

Table 2: Representation of $\left(\widehat{F}, A\right)$

U	e_1	e_2
(u_1)	$\langle [0.2, 0.7] ; [0.2, 0.5] ; [0.3, 0.8] \rangle$	$\langle [0.3, 0.9] ; [0.3, 0.5] ; [0.1, 0.7] \rangle$
(u_2)	$\langle [0.1, 0.3] ; [0.5, 0.7] ; [0.7, 0.9] \rangle$	$\langle [0.2, 0.8] ; [0.1, 0.9] ; [0.2, 0.8] \rangle$
(u_3)	$\langle [0.4, 0.8] ; [0.3, 0.6] ; [0.2, 0.6] \rangle$	$\langle [0.4, 0.5] ; [0, 0.8] ; [0.5, 0.6] \rangle$

The tabular representation of the NVSS $\left(\widehat{G}, B\right)$ is as in Table 3.

Table 3: Representation of $\left(\widehat{G}, B\right)$

U	e_1	e_2	e_3
(u_1)	$\langle[0.3,0.8];[0.1,0.4];[0.2,0.7]\rangle$	$\langle[0.3,1];[0.2,0.5];[0,0.7]\rangle$	$\langle[0.4,0.5];[0.2,0.3];[0.5,0.6]\rangle$
(u_2)	$\langle[0.6,0.9];[0.4,0.6];[0.1,0.3]\rangle$	$\langle[0.2,0.9];[0,0.2];[0.1,0.8]\rangle$	$\langle[0.3,0.7];[0.5,0.7];[0.3,0.7]\rangle$
(u_3)	$\langle[0.4,0.9];[0.3,0.6];[0.1,0.6]\rangle$	$\langle[0.5,0.6];[0,0.4];[0.4,0.5]\rangle$	$\langle[0.7,0.9];[0.7,0.8];[0.1,0.3]\rangle$

It is clear that $\left(\widehat{F}, A\right) \subseteq \left(\widehat{G}, B\right)$.

Definition 3.6 For a NVSS $\left(\widehat{F}, A\right)$ over a universe U, we say that $\left(\widehat{F}, A\right)$ is a null NVSS denoted by ψ_A , if $T_{\widehat{F}_{(e)}}(m) = [0,0]$, $I_{\widehat{F}_{(e)}}(m) = [1,1]$ and $F_{\widehat{F}_{(e)}}(m) = [1,1]$, $\forall\, m \in U$ and $\forall\, e \in A$.

Definition 3.7 For a NVSS $\left(\widehat{F}, A\right)$ over a universe U, we say that $\left(\widehat{F}, A\right)$ is an absolute NVSS denoted by $\widehat{\Psi}$, if $\forall\, m \in U$ and $\forall\, e \in A$, $T_{\widehat{F}_{(e)}}(m) = [1,1]$, $I_{\widehat{F}_{(e)}}(m) = [0,0]$ and $F_{\widehat{F}_{(e)}}(m) = [0,0]$.

4. Basic Operations

In this section, we define some basic operations of NVSS such as complement, union and intersection along with illustrative examples, followed by some properties .

We define the complement operation for NVSS and give an illustrative example.

Definition 4.1 Let $\left(\widehat{F}, A\right)$ be a NVSS over a universe U. Then the complement of $\left(\widehat{F}, A\right)$ is denoted by $\left(\widehat{F}, A\right)^c$ and is defined as $\left(\widehat{F}, A\right)^c = \left(\widehat{F}^c, A\right)$, where $\widehat{F}^c : A \to NV(U)$ is a mapping given by

$$\widehat{F}^c(\alpha) = \tilde{c}\ (\widehat{F}(\alpha)), \quad \forall \alpha \in A,$$

where $\tilde{c}$ is a neutrosophic vague complement.

Example 4.2 Consider Example 3.2, Then $\left(\widehat{F},A\right)^c$ is given by

$$\left(\widehat{F},A\right)^c = \Big\{\Big(e_1, \Big\{\frac{t_1}{\langle[0.2,0.8];[0.7,0.9];[0.2,0.8]\rangle}, \frac{t_2}{\langle[0.3,0.9];[0.5,0.8];[0.1,0.7]\rangle}, ,$$
$$\frac{t_3}{\langle[0.4,0.5];[0.3,0.7];[0.5,0.6]\rangle}, \frac{t_4}{\langle[0,0.2];[0.8,0.9];[0.8,1]\rangle}, \Big\}\Big),$$
$$\Big(e_2, \Big\{\frac{t_1}{\langle[0.1,0.2];[0.6,0.7];[0.8,0.9]\rangle}, \frac{t_2}{\langle[0.6,0.8];[0.6,0.8];[0.2,0.4]\rangle},$$
$$\frac{t_3}{\langle[0.5,1];[0.3,0.5];[0,0.5]\rangle}, \frac{t_4}{\langle[0.3,0.4];[0.6,0.8];[0.6,0.7]\rangle}, \Big\}\Big),$$
$$\Big(e_3, \Big\{\frac{t_1}{\langle[0.1,0.4];[0.6,0.8];[0.6,0.9]\rangle}, \frac{t_2}{\langle[0.2,0.3];[0.5,0.7];[0.7,0.8]\rangle},$$
$$\frac{t_3}{\langle[0.2,0.4];[0.6,0.9];[0.6,0.8]\rangle}, \frac{t_4}{\langle[0.6,0.8];[0.4,0.5];[0.2,0.4]\rangle}\Big\}\Big)\Big\}.$$

We will next define the union of two NVSSs and give an illustrative example.

Definition 4.3 Let $\left(\widehat{F},A\right)$ and $\left(\widehat{G},B\right)$ be two NVSSs over a universe U. Then the *union* of $\left(\widehat{F},A\right)$ and $\left(\widehat{G},B\right)$, denoted by $\left(\widehat{F},A\right)\tilde{\cup}\left(\widehat{G},B\right)$, and is defined as $\left(\widehat{F},A\right)\tilde{\cup}\left(\widehat{G},B\right) = \left(\widehat{H},C\right)$ such that $C = A\cup B$ and $\forall\, e \in C$, is

$$(\widehat{H},C) = \begin{cases} \widehat{F}(e) & ,\text{if } e \in A - B \\ \widehat{G}(e) & ,\text{if } e \in B - A \\ \widehat{F}(e)\tilde{\cup}\widehat{G}(e) & ,\text{if } e \in A\cap B, \end{cases}$$

where $\tilde{\cup}$ denotes the neutrosophic vague set union.

Example 4.4 Consider Example 3.2, $\left(\widehat{F},A\right)$ and $\left(\widehat{G},B\right)$ are two NVSSs with tabular representations as in Table 4 and Table 5.

Table 4: Representation of $\left(\widehat{F},A\right)$

U	$e_1 = Large$	$e_2 = Medium$	$e_3 = Small$
(t_1)	$\langle[0.2,0.8];[0.1,0.3];[0.2,0.8]\rangle$	$\langle[0.8,0.9];[0.3,0.4];[0.1,0.2]\rangle$	$\langle[0.6,0.9];[0.2,0.4];[0.1,0.4]\rangle$
(t_2)	$\langle[0.1,0.7];[0.2,0.5];[0.3,0.9]\rangle$	$\langle[0.2,0.4];[0.2,0.4];[0.6,0.8]\rangle$	$\langle[0.7,0.8];[0.3,0.5];[0.2,0.3]\rangle$
(t_3)	$\langle[0.5,0.6];[0.3,0.7];[0.4,0.5]\rangle$	$\langle[0,0.5];[0.5,0.7];[0.5,1]\rangle$	$\langle[0.6,0.8];[0.1,0.4];[0.2,0.4]\rangle$
(t_4)	$\langle[0.8,1];[0.1,0.2];[0,0.2]\rangle$	$\langle[0.6,0.7];[0.2,0.4];[0.3,0.4]\rangle$	$\langle[0.2,0.4];[0.5,0.6];[0.6,0.8]\rangle$

Table 5: Representation of $\left(\widehat{G}, B\right)$

U	$e_1 = Small$	$e_2 = X - Large$
(t_1)	$\langle [0.2, 0.5]\,;[0.1, 0.6]\,;[0.5, 0.8] \rangle$	$\langle [0.5, 0.6]\,;[0.4, 0.8]\,;[0.4, 0.5] \rangle$
(t_2)	$\langle [0.6, 0.7]\,;[0.2, 0.7]\,;[0.2, 0.4] \rangle$	$\langle [0.4, 0.7]\,;[0.2, 0.8]\,;[0.3, 0.6] \rangle$
(t_3)	$\langle [0.1, 0.2]\,;[0.5, 0.9]\,;[0.8, 0.9] \rangle$	$\langle [0.7, 0.9]\,;[0.2, 0.3]\,;[0.1, 0.3] \rangle$
(t_4)	$\langle [0.3, 0.4]\,;[0.1, 0.4]\,;[0.6, 0.7] \rangle$	$\langle [0.2, 0.4]\,;[0.7, 0.9]\,;[0.6, 0.8] \rangle$

The union of $\left(\widehat{F}, A\right)$ and $\left(\widehat{G}, B\right)$ can be represented as in Table 6.

Table 6: $\left(\widehat{F}, A\right) \widetilde{\cup} \left(\widehat{G}, B\right)$

U	$e_1 = Large$	$e_2 = Medium$
(t_1)	$\langle [0.2, 0.8]\,;[0.1, 0.3]\,;[0.2, 0.8] \rangle$	$\langle [0.8, 0.9]\,;[0.3, 0.4]\,;[0.1, 0.2] \rangle$
(t_2)	$\langle [0.1, 0.7]\,;[0.2, 0.5]\,;[0.3, 0.9] \rangle$	$\langle [0.2, 0.4]\,;[0.2, 0.4]\,;[0.6, 0.8] \rangle$
(t_3)	$\langle [0.5, 0.6]\,;[0.3, 0.7]\,;[0.4, 0.5] \rangle$	$\langle [0, 0.5]\,;[0.5, 0.7]\,;[0.5, 1] \rangle$
(t_4)	$\langle [0.8, 1]\,;[0.1, 0.2]\,;[0, 0.2] \rangle$	$\langle [0.6, 0.7]\,;[0.2, 0.4]\,;[0.3, 0.4] \rangle$
U	$e_3 = Small$	$e_4 = X - Large$
(t_1)	$\langle [0.6, 0.9]\,;[0.1, 0.4]\,;[0.1, 0.4] \rangle$	$\langle [0.5, 0.6]\,;[0.4, 0.8]\,;[0.4, 0.5] \rangle$
(t_2)	$\langle [0.7, 0.8]\,;[0.2, 0.5]\,;[0.2, 0.3] \rangle$	$\langle [0.4, 0.7]\,;[0.2, 0.8]\,;[0.3, 0.6] \rangle$
(t_3)	$\langle [0.6, 0.8]\,;[0.1, 0.4]\,;[0.2, 0.4] \rangle$	$\langle [0.7, 0.9]\,;[0.2, 0.3]\,;[0.1, 0.3] \rangle$
(t_4)	$\langle [0.3, 0.4]\,;[0.1, 0.4]\,;[0.6, 0.7] \rangle$	$\langle [0.2, 0.4]\,;[0.7, 0.9]\,;[0.6, 0.8] \rangle$

We introduce the definition of intersection operation, give an illustrative example and derive some properties of union and intersection operations.

Definition 4.5 Let $\left(\widehat{F}, A\right)$ and $\left(\widehat{G}, B\right)$ be two NVSSs over a universe U. Then the *intersection* of $\left(\widehat{F}, A\right)$ and $\left(\widehat{G}, B\right)$, denoted by $\left(\widehat{F}, A\right) \widetilde{\cap} \left(\widehat{G}, B\right)$, and is defined as $\left(\widehat{F}, A\right) \widetilde{\cap} \left(\widehat{G}, B\right) = \left(\widehat{K}, C\right)$ such that $C = A \cup B$ and $\forall\, e \in C$, is

$$(\widehat{K}, C) = \begin{cases} \widehat{F}(e) & ,\text{if } e \in A - B \\ \widehat{G}(e) & ,\text{if } e \in B - A \\ \widehat{F}(e) \widetilde{\cap} \widehat{G}(e) & ,\text{if } e \in A \cap B, \end{cases}$$

where $\widetilde{\cap}$ denotes the neutrosophic vague set intersection.

Example 4.6 Consider Example 4.4. By using basic neutrosophic vague set intersection we can easily verify that $\left(\widehat{F}, A\right) \widetilde{\cap} \left(\widehat{G}, B\right) = \left(\widehat{K}, C\right)$, where the tabular representation of $\left(\widehat{K}, C\right)$ is as in Table 7.

Table 7: $\left(\widehat{F}, A\right) \widetilde{\cap} \left(\widehat{G}, B\right)$

U	$e_1 = Large$	$e_2 = Medium$
(t_1)	$\langle [0.2, 0.8] ; [0.1, 0.3] ; [0.2, 0.8] \rangle$	$\langle [0.8, 0.9] ; [0.3, 0.4] ; [0.1, 0.2] \rangle$
(t_2)	$\langle [0.1, 0.7] ; [0.2, 0.5] ; [0.3, 0.9] \rangle$	$\langle [0.2, 0.4] ; [0.2, 0.4] ; [0.6, 0.8] \rangle$
(t_3)	$\langle [0.5, 0.6] ; [0.3, 0.7] ; [0.4, 0.5] \rangle$	$\langle [0, 0.5] ; [0.5, 0.7] ; [0.5, 1] \rangle$
(t_4)	$\langle [0.8, 1] ; [0.1, 0.2] ; [0, 0.2] \rangle$	$\langle [0.6, 0.7] ; [0.2, 0.4] ; [0.3, 0.4] \rangle$
U	$e_3 = Small$	$e_4 = X - Large$
(t_1)	$\langle [0.2, 0.5] ; [0.2, 0.6] ; [0.5, 0.8] \rangle$	$\langle [0.5, 0.6] ; [0.4, 0.8] ; [0.4, 0.5] \rangle$
(t_2)	$\langle [0.6, 0.7] ; [0.3, 0.7] ; [0.2, 0.4] \rangle$	$\langle [0.4, 0.7] ; [0.2, 0.8] ; [0.3, 0.6] \rangle$
(t_3)	$\langle [0.1, 0.2] ; [0.5, 0.9] ; [0.8, 0.9] \rangle$	$\langle [0.7, 0.9] ; [0.2, 0.3] ; [0.1, 0.3] \rangle$
(t_4)	$\langle [0.2, 0.4] ; [0.5, 0.6] ; [0.6, 0.8] \rangle$	$\langle [0.2, 0.4] ; [0.7, 0.9] ; [0.6, 0.8] \rangle$

Proposition 4.7 For any two NVSSs $\left(\widehat{H}, A\right)$ and $\left(\widehat{K}, B\right)$ over a universe U, we have :

1. Idempotency Laws:
   a. $\left(\widehat{H}, A\right) \widetilde{\cup} \left(\widehat{H}, A\right) = \left(\widehat{H}, A\right)$ b. $\left(\widehat{H}, A\right) \widetilde{\cap} \left(\widehat{H}, A\right) = \left(\widehat{H}, A\right)$.
2. Commutative Laws:
   a. $\left(\widehat{H}, A\right) \widetilde{\cup} \left(\widehat{K}, B\right) = \left(\widehat{K}, B\right) \widetilde{\cup} \left(\widehat{H}, A\right)$.
   b. $\left(\widehat{H}, A\right) \widetilde{\cap} \left(\widehat{K}, B\right) = \left(\widehat{K}, B\right) \widetilde{\cap} \left(\widehat{H}, A\right)$.

Proof. The proof is straightforward and thus omitted.

Proposition 4.8 For any three NVSSs $\left(\widehat{H}, A\right)$,$\left(\widehat{K}, B\right)$ and $\left(\widehat{L}, C\right)$ over U, we have:

1. $(\widehat{H}, A) \widetilde{\cup} \left((\widehat{K}, B) \widetilde{\cup} (\widehat{L}, C)\right) = \left((\widehat{H}, A) \widetilde{\cup} (\widehat{K}, B)\right) \widetilde{\cup} (\widehat{L}, C)$,

2. $(\widehat{H},A)\widetilde{\cap}\left((\widehat{K},B)\widetilde{\cap}(\widehat{L},C)\right)=\left((\widehat{H},A)\widetilde{\cap}(\widehat{K},B)\right)\widetilde{\cap}(\widehat{L},C)$,

3. $(\widehat{H},A)\widetilde{\cap}\left((\widehat{K},B)\widetilde{\cup}(\widehat{L},C)\right)=\left((\widehat{H},A)\widetilde{\cap}(\widehat{K},B)\right)\widetilde{\cup}\left((\widehat{H},A)\widetilde{\cap}(\widehat{L},C)\right)$,

4. $(\widehat{H},A)\widetilde{\cup}\left((\widehat{K},B)\widetilde{\cap}(\widehat{L},C)\right)=\left((\widehat{H},A)\widetilde{\cup}(\widehat{K},B)\right)\widetilde{\cap}\left((\widehat{H},A)\widetilde{\cup}(\widehat{L},C)\right)$.

Proof. The proof is straightforward .

5. An Adjustable Approach to NVSSs Based Decision Making

We will now present an adjustable approach to NVSSs based decision making problems by extending the approach to interval-valued intuitionistic fuzzy soft sets based decision making (Zhang et al. (2014)).

5.1 Level soft sets of NVSSs

We present here an adjustable approach to NVSS based decision making problems. This proposal is based on the following novel concept called level soft sets.

Definition 5.1 Denote $L=\{(\alpha,\beta,\gamma)|\ \alpha=[\alpha_1,\alpha_2]\in Int([0,1]),\ \beta=[\beta_1,\beta_2]\in Int([0,1]),\ \gamma=[\gamma_1,\gamma_2]\in Int([0,1]),\ 0\leq\alpha_2+\beta_2+\gamma_2\leq 2\}$, where $Int([0,1])$ denotes the set of all closed subintervals of [0,1]. We define a relation $\geq_L$ on L as follows :
for all $(\alpha,\beta,\gamma),\ (\zeta,\eta,\theta)\in L,\ (\alpha,\beta,\gamma)\geq_L(\zeta,\eta,\theta)\Longleftrightarrow\alpha\geq\zeta,\ \beta\leq\eta,\ and\ \gamma\leq\theta\Longleftrightarrow[\alpha_1,\ \alpha_2]\geq[\zeta_1,\ \zeta_2],\ [\beta_1,\ \beta_2]\leq[\eta_1,\ \eta_2],\ [\gamma_1,\ \gamma_2]\leq[\theta_1,\ \theta_2]\Longleftrightarrow\alpha_1\geq\zeta_1,\ \alpha_2\geq\zeta_2,\ \beta_1\leq\eta_1,\ \beta_2\leq\eta_2,\ \gamma_1\leq\theta_1,\ \gamma_2\leq\theta_2$.

Definition 5.2 Let $\varpi=(\widehat{F},A)$ be a NVSS over a universe U, E a set of parameters and $A\subseteq E$. For $(\alpha,\beta,\gamma)\in L$, the (α,β,γ)-level soft set of ϖ is a crisp soft set $L(\varpi;\alpha,\beta,\gamma)=(\widehat{F}_{(\alpha,\beta,\gamma)},A)$ defined by
$\widehat{F}_{(\alpha,\beta,\gamma)}(e)=L(\widehat{F}_{(e)};\alpha,\beta,\gamma)=\{x\in U|\ \widehat{F}_{(e)}(x)\geq_L(\alpha,\beta,\gamma)\}=\{x\in U|\ T_{\widehat{F}(e)}(x)\geq\alpha,\ I_{\widehat{F}(e)}(x)\leq\beta,\ F_{\widehat{F}(e)}(x)\leq\gamma\}$ for all $e\in A$.

5.2 Level soft sets with respect to a threshold neutrosophic vague set

In Definition 5.2 the level triple (or threshold triple) assigned to each parameter is always a constant value triple $(\alpha, \beta, \gamma) \in L$. However, in some practical applications, decision makers need to impose different thresholds on different parameters. To address this issue, we replace the constant value triple by a function as the thresholds on truth-membership values, indeterminacy-membership values and falsity-membership values, respectively.

Definition 5.3 Let $\varpi = (\widehat{F}, A)$ be a NVSS over U, where $A \subseteq E$ and E is a parameter set. Let $\lambda : A \to [0,1] \times [0,1] \times [0,1]$ be a neutrosophic vague set in A which is called a threshold neutrosophic vague set. The level soft set of ϖ with respect to λ is a crisp soft set $L(\varpi; \lambda) = (\widehat{F}_\lambda, A)$ defined by
$\widehat{F}_\lambda(e) = L(\widehat{F}(e); \lambda(e)) = \{x \in U |\ \widehat{F}_e(x) \geq_L \lambda(e)\} = \{x \in U |\ T_{\widehat{F}(e)}(x) \geq T_\lambda(e), I_{\widehat{F}(e)}(x) \leq I_\lambda(e)$ and $F_{\widehat{F}(e)}(x) \leq F_\lambda(e)\}$ for all $e \in A$.

It is clear that level soft sets with respect to a neutrosophic vague set generalize (α, β, γ) - level soft sets by substituting a function on the parameter set A, namely a neutrosophic vague set $\lambda : A \to [0,1] \times [0,1] \times [0,1]$, for constants $(\alpha, \beta, \gamma) \in [0,1] \times [0,1] \times [0,1]$.

In order to better understand the above idea, consider the following implementations.

Example 5.4 Let $\varpi = (\widehat{F}, A)$ be a NVSS over a universe U, E a set of parameters and $A \subseteq E$. Based on the NVSS $\varpi = (\widehat{F}, A)$, a neutrosophic vague set $avg_\varpi : A \to [0,1] \times [0,1] \times [0,1]$ can be defined as:

$T_{avg\varpi_L}(e) = \frac{1}{|U|} \sum\limits_{x \in U} T_{\widehat{F}(e)_L}(x)$, $T_{avg\varpi_R}(e) = \frac{1}{|U|} \sum\limits_{x \in U} T_{\widehat{F}(e)_R}(x)$, $I_{avg\varpi_L}(e) = \frac{1}{|U|} \sum\limits_{x \in U} I_{\widehat{F}(e)_L}(x)$, $I_{avg\varpi_R}(e) = \frac{1}{|U|} \sum\limits_{x \in U} I_{\widehat{F}(e)R}(x)$, $F_{avg\varpi_L}(e) = \frac{1}{|U|} \sum\limits_{x \in U} F_{\widehat{F}(e)_L}(x)$, $F_{avg\varpi_R}(e) = \frac{1}{|U|} \sum\limits_{x \in U} F_{\widehat{F}(e)_R}(x)$.

The neutrosophic vague set $avg\varpi$ is called the avg - threshold of the NVSS ϖ . In addition, the level soft set of ϖ with respect to the avg - threshold neutrosophic vague set avg ϖ, namely $L(\varpi; avg\varpi)$ is called the avg - level soft set of ϖ and can be simply denoted by $L(\varpi; avg) = (\widehat{F}_{avg_\varpi}, A)$ and it can be defined as follows:

$\widehat{F}_{avg_\varpi}(e) = L(\widehat{F}(e); avg_\varpi(e)) = \{x \in U|\ \widehat{F}_e(x) \geq_L avg_\varpi(e)\} = \{x \in U|\ T_{\widehat{F}(e)}(x) \geq T_{avg_\varpi}(e), I_{\widehat{F}(e)}(x) \leq I_{avg_\varpi}(e)$ and $F_{\widehat{F}(e)}(x) \leq F_{avg_\varpi}(e)\}$, for all $e \in A$.

For a concrete example of avg - level soft sets, reconsider the NVSS $\varpi = (\widehat{F}, A)$ with its tabular representation given by Table 8 in the following example.

Example 5.5 Suppose that a customer wants to select a house from a real estate agent. He can construct a NVSS $\varpi = (\widehat{F}, A)$ that describes the characteristic of houses according to his preference list. Assume that $U = \{u_1, u_2, u_3, u_4\}$ is the universe containing four houses under consideration and $E = \{e_1 = cheap,\ e_2 = beautiful,\ e_3 =$ green surrounding, $e_4 =$ spacious $\}$.

Suppose that

$$\widehat{F}(e_1) = \Big\{ \frac{u_1}{\langle[0.5,0.7];[0.1,0.4];[0.3,0.5]\rangle}, \frac{u_2}{\langle[0.9,1];[0.2,0.3];[0,0.1]\rangle}, \frac{u_3}{\langle[0.1,0.2];[0.4,0.5];[0.8,0.9]\rangle}, \frac{u_4}{\langle[0.5,0.8];[0.1,0.3];[0.2,0.5]\rangle} \Big\},$$

$$\widehat{F}(e_2) = \Big\{ \frac{u_1}{\langle[0.4,0.7];[0.2,0.5];[0.3,0.6]\rangle}, \frac{u_2}{\langle[0.2,0.5];[0.1,0.3];[0.5,0.8]\rangle}, \frac{u_3}{\langle[0.8,0.9];[0.5,0.6];[0.1,0.2]\rangle}, \frac{u_4}{\langle[0.4,0.7];[0.4,0.5];[0.3,0.6]\rangle} \Big\},$$

$$\widehat{F}(e_3) = \Big\{ \frac{u_1}{\langle[0.6,0.8];[0.1,0.3];[0.2,0.4]\rangle}, \frac{u_2}{\langle[0.3,0.4];[0.5,0.6];[0.6,0.7]\rangle}, \frac{u_3}{\langle[0.7,0.8];[0.3,0.4];[0.2,0.3]\rangle}, \frac{u_4}{\langle[0.6,0.7];[0.1,0.4];[0.3,0.4]\rangle} \Big\},$$

$$\widehat{F}(e_4) = \Big\{ \frac{u_1}{\langle[0.5,0.6];[0.3,0.4];[0.4,0.5]\rangle}, \frac{u_2}{\langle[0.4,0.5];[0.2,0.4];[0.5,0.6]\rangle}, \frac{u_3}{\langle[0.6,0.9];[0.2,0.5];[0.1,0.4]\rangle}, \frac{u_4}{\langle[0.5,0.8];[0.2,0.4];[0.2,0.5]\rangle} \Big\}.$$

Table 8 gives the tabular representation of the NVSS $\varpi = (\widehat{F}, A)$.

Table 8: Tabular form of the NVSS $(\widehat{F}, A)$

U	e_1	e_2
(u_1)	$\langle [0.5, 0.7]\,;[0.1, 0.4]\,;[0.3, 0.5] \rangle$	$\langle [0.4, 0.7]\,;[0.2, 0.5]\,;[0.3, 0.6] \rangle$
(u_2)	$\langle [0.9, 1]\,;[0.2, 0.3]\,;[0, 0.1] \rangle$	$\langle [0.2, 0.5]\,;[0.1, 0.3]\,;[0.5, 0.8] \rangle$
(u_3)	$\langle [0.1, 0.2]\,;[0.4, 0.5]\,;[0.8, 0.9] \rangle$	$\langle [0.8, 0.9]\,;[0.5, 0.6]\,;[0.1, 0.2] \rangle$
(u_4)	$\langle [0.5, 0.8]\,;[0.1, 0.3]\,;[0.2, 0.5] \rangle$	$\langle [0.4, 0.7]\,;[0.4, 0.5]\,;[0.3, 0.6] \rangle$
U	e_3	e_4
(u_1)	$\langle [0.6, 0.8]\,;[0.1, 0.3]\,;[0.2, 0.4] \rangle$	$\langle [0.5, 0.6]\,;[0.3, 0.4]\,;[0.4, 0.5] \rangle$
(u_2)	$\langle [0.3, 0.4]\,;[0.5, 0.6]\,;[0.6, 0.7] \rangle$	$\langle [0.4, 0.5]\,;[0.2, 0.4]\,;[0.5, 0.6] \rangle$
(u_3)	$\langle [0.7, 0.8]\,;[0.3, 0.4]\,;[0.2, 0.3] \rangle$	$\langle [0.6, 0.9]\,;[0.2, 0.5]\,;[0.1, 0.4] \rangle$
(u_4)	$\langle [0.6, 0.7]\,;[0.1, 0.4]\,;[0.3, 0.4] \rangle$	$\langle [0.5, 0.8]\,;[0.2, 0.4]\,;[0.2, 0.5] \rangle$

We can calculate the avg - threshold of $\varpi = (\widehat{F}, A)$ as follows:

$avg_{(\widehat{F},A)} = \{\langle [0.5, 0.67]; [0.2, 0.38]; [0.33, 0.5] \backslash e_1 \rangle, \langle [0.45, 0.7]; [0.3, 0.48]; [0.3, 0.55] \backslash e_2 \rangle, \langle [0.55, 0.67]; [0.25, 0.43]; [0.33, 0.45] \backslash e_3 \rangle, \langle [0.5, 0.7]; [0.23, 0.43]; [0.3, 0.5] \backslash e_4 \rangle\}$.

The avg - level soft set of $\varpi = (\widehat{F}, A)$ is $L(\varpi; avg) = (\widehat{F}_{avg_{\varpi}}, A)$ and it can be calculated as follows:

$\widehat{F}_{avg_{\varpi}}(e_1) = L(\widehat{F}(e_1); avg_{\varpi}(e_1)) = \{u_2, u_4\}$, $F_{avg_{\varpi}}(e_2) = L(\widehat{F}(e_2); avg_{\varpi}(e_2)) = \phi$, $\widehat{F}_{avg_{\varpi}}(e_3) = L(\widehat{F}(e_3); avg_{\varpi}(e_3)) = \{u_1, u_4\}$, $\widehat{F}_{avg_{\varpi}}(e_4) = L(\widehat{F}(e_4); avg_{\varpi}(e_4)) = \{u_4\}$.

5.3 An adjustable approach based on level soft sets

Now, we construct a NVSS set decision making method by the following algorithm .

Algorithm 1.

1. Input the (resultant) NVSS $\varpi = (\widehat{F}, A)$

2. Input a threshold neutrosophic vague set $\lambda : A \to [0,1] \times [0,1] \times [0,1]$ (or give a threshold value triple $(\alpha, \beta, \gamma) \in [0,1] \times [0,1] \times [0,1]$; or choose the avg-level decision rule) for decision making.

3. Compute the level soft set $L(\varpi, \lambda)$ with respect to the threshold neutrosophic vague set λ (or the $(\alpha, \beta, \gamma)-$ level soft set $L(\varpi; \alpha, \beta, \gamma)$; or the avg-level soft set $L(\varpi; avg)$).

4. Present the level soft set $L(\varpi, \lambda)(or L(\varpi; \alpha, \beta, \gamma); or L(\varpi; avg))$ in tabular form .

5. Compute the choice value c_i of u_i for any $u_i \in U$.

6. The optimal decision is to select u_k if $c_k = max_{u_i \in U}(c_i)$.

7. If k has more than one value then any one of u_k may be chosen.

Remark 5.6 In the last step of the above algorithm, one may go back to the second step and change the threshold (or decision rule) that was used so as to adjust the final optimal decision, especially when there are too many "optimal choices" to be chosen.

Example 5.7 Assume that $\varpi = (\widehat{F}, A)$ is a NVSS with its tabular representation shown in Table 8. If we deal with the decision making problem involving $\varpi = (\widehat{F}, A)$ by avg-level decision rule, we shall use the avg-threeshold $avg_{(\widehat{F},A)}$ and obtain the avg-level soft set $L((\widehat{F}, A); avg)$. Table 9 represents the avg-level soft set $L((\widehat{F}, A); avg)$. If $u_i \in \widehat{F}_{avg_{\varpi}}(e_j)$, then $u_{ij} = 1$, otherwise $u_{ij} = 0$, where u_{ij} are the entries in Table 9, for all $i, j = \{1, 2, 3, 4\}$.

Table 9: $L((\widehat{F}, A); avg)$

U	e_1	e_2	e_3	e_4
u_1	0	0	1	0
u_2	1	0	0	0
u_3	0	0	0	0
u_4	1	0	1	1

Table 10 represents $L((\widehat{F}, A); avg)$ with choice values .

Table 10: $L((\widehat{F}, A); avg)$ with choice value

U	e_1	e_2	e_3	e_4	choice value (c_i)
u_1	0	0	1	0	$c_1 = 1$
u_2	1	0	0	0	$c_2 = 1$
u_3	0	0	0	0	$c_3 = 0$
u_4	1	0	1	1	$c_4 = 3$

We can calculate the choice values from Table 10.

$$c_1 = \sum_{j=1}^{4} u_{1j} = 0 + 0 + 1 + 0 = 1.$$

Similarly, we have $c_2 = 1,\ c_3 = 0,\ and\ c_4 = 3$.

The optimal decision is to select the house u_4, since $c_4 = max_{u_i \in U}(c_i)$.

5.4 Comparison between NVSS to other existing methods

In this section, we will compare our proposed NVSS model to two other existing models, the vague soft set (Xu et al. (2010)) and neutrosophic soft set (Maji (2013)).

The vague soft set is actually a generalization of soft set (Molodtsov (1999)) and fuzzy soft set (Maji et al. (2001a)), while the neutrosophic soft set is actually a generalization of soft set, fuzzy soft set and intuitionistic fuzzy soft set (Maji et al. (2001b)). To reveal the significance of our proposed NVSS compared to vague soft set, let us consider Example 5.5 above.

For example, the approximation $\widehat{F}(e_1)$ can be presented by vague soft set as follows.

$$\widehat{F}(e_1) = \left\{ \frac{u_1}{\langle[0.5,0.7]\rangle}, \frac{u_2}{\langle[0.9,1]\rangle}, \frac{u_3}{\langle[0.1,0.2]\rangle}, \frac{u_4}{\langle[0.5,0.8]\rangle} \right\}.$$

Note that the NVSS is a generalization of vague soft set. Thus as shown in Example 5.5 above, the NVSS can explain the universal U in more detail with three membership functions, whereas vague soft set can tell us a limited information about the universal U. It can only handle the incomplete information considering both the truth-membership and falsity-membership values, while NVSS can handle problems involving imprecise, indeterminacy and inconsistent data, which makes it more accurate and realistic than vague soft set.

It is worthy to note that NVSS is more advantageous than neutrosophic soft set by virtue of vague set which allows using interval-based membership instead of using point-based membership as in neutrosophic set. This enables NVSS to better capture the vagueness and uncertainties of the data which is prevalent in most real-life situations.

6. Conclusion

We established the concept of neutrosophic vague soft set by applying the theory of soft set (Molodtsov (1999)) to neutrosophic vague set (Alkhazaleh (2015)). The basic operations on neutrosophic vague soft set, namely complement, subset, union, intersection operations, were defined along with several examples. Subsequently, the basic properties of these operations were proven . Finally, a generalized algorithm is introduced and applied to neutrosophic vague soft set model to solve a hypothetical decision making problem. This new extension will provide a significant addition to existing theories for handling problems involving imprecise, indeterminacy and inconsistent data, and spurs more developments of further research and pertinent applications.

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